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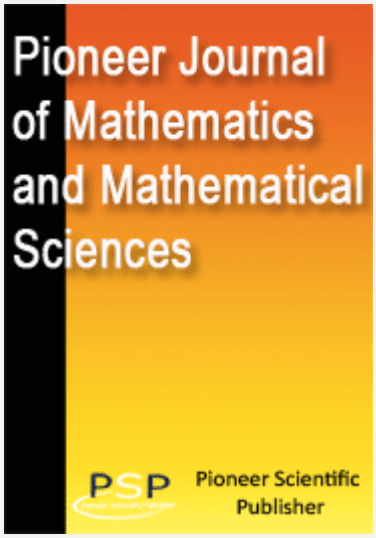

INSURANCE SOLVENCY RISK ANALYTICS: COST-OF-CAPITAL AND PROSPECTIVE LIABILITY APPROACH

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Abstract

Within regulatory solvency systems for insurance companies the quantification of the insurance risk and market risk components of the required solvency risk capital is of primordial importance. Defining the random insurance loss over a given discrete time period as the unexpected decrease of the risk bearing capital over that period given the available information, and using Doob's decomposition theorem for the risk bearing capital, we show that the insurance losses form a sequence of uncorrelated random variables with vanishing mean (Theorem of Hattendorff). General formulas for the target capital (sum of economic capital and risk margin) according to the cost-of-capital and the prospective liability approach are derived. The prospective liability approach is used to determine the insurance risk related solvency capital (e.g., the biometric risk solvency capital) while the cost-of-capital approach is used to determine the market risk related component of it. We derive closed-form formulas for the mean and variance of the random future risk-bearing capital under a multivariate lognormal model of the investment return and restate them for the special case of the Black-Scholes-Merton return model. Under a constant expected discount rate and deterministic future premium loadings, we derive formulas for the mean and variance of the discounted risk bearing capital. Under the further assumption of normally distributed discounted insurance losses, we obtain simple analytical formulas for the market risk target capital at initial time for any type of insurance business. Numerical results are tabulated, interpreted and compared.

Keywords and phrases: solvency risk, insurance risk, market risk, cost-of-capital approach, prospective liability approach, lognormal returns, insurance loss normal approximation.